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New Geometrical Model of Woven Fabric Taking into Account the Change of Its Form, Size and Lateral Bending

Abstract
This paper suggests a technique for the construction of fabric three-dimensional models based on the piecewise continuous representation of the thread surface. The approach allows to exclude such simulation defects as "interpenetration" of threads in the fabric, and takes into account the geometric features of the thread structure, the possibility of their bending in several planes, and changes in the form and sectional area. The geometric models obtained can be used to analyse the forces and deformations occurring in the fabric with the help of the finite-element simulation programs in various specialised software complexes, such as ANSYS, LS-DYNA, ABAQUS, NASTRAN etc. The structure parameters which are difficult to calculate using standard techniques, e.g. the pore volume and contact area between threads can be calculated with the help of describing the fabric three-dimensional structure.

Key words: *fabric, 3D simulation, warp, filling, bending waves' height, fabric structure.*

RESEARCH & DEVELOPMENT

Introduction

Three-dimensional fabric simulation allows to understand the fabric structure, which is especially important when designing complex structure fabrics (multilayered fabrics, fabrics with variable density of threads etc.). The structure parameters which are difficult to calculate using standard techniques (pore volume, contact area between threads etc.) can be calculated with the help of mathematical models describing the fabric three-dimensional structure. The fabric 3D models can be used to calculate forces and deformations arising in the fabric with the help of finite-element simulation programs in various specialised software complexes, for example, ANSYS, LS-DYNA, ABAQUS, NASTRAN [1-3], and others. The advantage of 3D models of threads and textile material is the ability to take into account the threads bending in different directions, which allows more accurate prediction of many parameters, for example, fabrics with large thread overlaps have lateral thread bending, which affects many structural indicators of the product form [4].

The traditional approach to thread simulation in a fabric structure is to calculate the middle line, and then the section curve sweeps along the yarn path to form sets of section planes [5]. The axial line or its part can be depicted as a sinusoid [6-8], as well as other trigonometric functions [9]. Also, the thread form can be specified in the form of splines [10-14]. The disadvantage of such approaches is the effect of thread "interpenetration" in the fabric due to approximations in the geometric models. For this, the authors suggest using a special mechanism for the elimination of such a phenomenon, which complicates the algorithm for the construction of a three-dimensional model.

This paper proposes the use of direct simulation of the thread surface in a fabric based on the piecewise linear representation of functions, which was described in paper [15]. Such an approach allows not only to exclude thread "interpenetration", but also to take into account the thread bending in several planes when simulating, as well as the change in the cross-sectional form of

threads caused by their deformation at intersections, which is associated with their unevenness.

With an accurate 3D model of the thread, it is possible to model complex phenomena, such as the recovery behaviour of fabrics [16] and the influence of weft density on fabric dynamic thickness under tensile forces [17].

Thread base form simulation

This paper presents a technique for the construction of a thread 3D model for a fabric element based on previously obtained data on the calculation of a thread profile in the form of the piecewise continuous function. The primary parameters of the structure can be obtained using the methods described, for example, in the work [18-19]. Then it is necessary to simulate the 3D structure of the thread in the fabric.

The function determining the position of the centre thread line *f(x)* is calculated based on the possible limits of the thread

$$
\begin{cases}
\left(\dfrac{d_{yv}}{2} + \dfrac{d_{ov}}{2}\right) \cdot \sqrt{1 - \dfrac{x^2}{\left(\frac{d_{yg}}{2} + \frac{d_{og}}{2}\right)^2}} + H, & if\ 0 < x \le \dfrac{x_1 + x_2}{2} \\[4ex]
d \cdot x + \dfrac{b + b_1}{2}, & if\ \dfrac{x_1 + x_3}{2} < x \le \dfrac{x_2 + x_4}{2} \\[4ex]
-\left(\dfrac{d_{yv}}{2} + \dfrac{d_{ov}}{2}\right) \cdot \sqrt{1 - \dfrac{(x - l_y)^2}{\left(\frac{d_{yg}}{2} + \frac{d_{og}}{2}\right)^2}} + E, & if\ \dfrac{x_2 + x_4}{2} < x \le l_y
\end{cases}
\tag{1}
$$

Equation (1).

profile. Then, if the thread cross-section is an ellipse, *Equation (1)* can be written.

Where, d_{yv} – weft vertical diameter; d_{yg} – weft horizontal diameter; d_{ov} –warp vertical diameter; H – transverse centre of the first weft thread; E – transverse centre of the second weft thread; l_y – distance between weft threads; x_1, x_2, x_3, x_4 – coordinates for the tangency points of the straight lines and curves describing the upper and lower limits of the thread profiles (the x-axis); b, b_1 – terms in the equation of the straight line connecting the curves that describe the limits of the thread profile in the fabric; d – slope of the straight line connecting the curves that describe the limits of the thread profile in the fabric.

Then, assuming that the thread takes the form of a curved elliptical cylinder (a closed cylindrical surface with any base can be used), we obtain the function $G_v(x, y)$, expressing the upper volumetric (in "*xyz*" coordinates) part of the thread form, see *Equation (2)*.

Equation (2), where, $j_v(x)$ –function describing the top of the thread profile; f(x) – function describing the middle line of the thread (with a bending in one plane); r – half of the warp horizontal diameter.

The lower part of the thread surface $G_n(x, y)$ is presented by *Equation (3)*.

Equation (3), where, $j_n(x)$ – function describing the lower part of the thread profile in space.

The parameters of the middle thread line are not necessary to be calculated, and then the upper and lower volume parts of the thread profile can be expressed as *Equations (4)* and *(5)*.

There is an example of function parameter calculation for the construction of the main thread three-dimensional model for linen fabric, a prototype of a plain structure at one bending interval. The fabric is characterised by the following parameters: weft density – 150 threads/10 cm, warp density – 170 threads/10 cm, fabric warp and weft diameters with regard to deformation – 0.274 mm vertically and 0.372 mm horizontally, height of the bending warp wave – 0.448 mm.

Piecewise continuous function describing the upper limits of the thread profile in the fabric [16], see *Equation (6)*.

Piecewise continuous function describing the lower limits of the thread profile in the fabric, see *Equation (7)*.

The three-dimensional model of this area can be expressed by the functions given as *Equations (8)* and *(9)*.

Figure 1 shows two surfaces constructed by *Equations (8)* and *(9)* taking into account the results of *Equations (6)* and *(7)*, describing the thread structure in the fabric at the bending interval.

A set of surfaces to describe the structure of a fabric in a three-dimensional form can be built with the help of this technique. An example of such a construction is shown in *Figure 2*. The primary parameters of the structure were calculated with the models described in work [18]. It can also be presented as a function of the number of fibres in the yarn cross-section and the packing density of fibres.
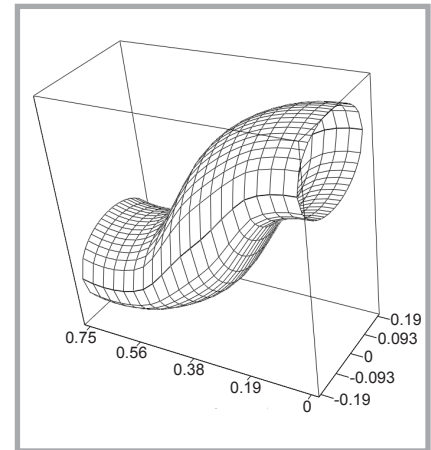

Figure 1. *Thread three-dimensional model for a fabric element.*

Horizontal resizing of the thread requires other calculation techniques, since the area for the initial value setting is not rectangular. The methods of parametric surface specification are better to be used for this.

$$G_v(x,y) = f(x) + (j_v(x) - f(x)) \cdot \sqrt{1 - \frac{y^2}{r^2}} \tag{2}$$

$$G_n(x,y) = f(x) + f(x) - (j_n(x)) \cdot \sqrt{1 - \frac{y^2}{r^2}} \tag{3}$$

$$G_v(x,y) = \frac{j_v(x) + j_n(x)}{2} + \left(\frac{j_v(x) - j_n(x)}{2}\right) \cdot \sqrt{1 - \frac{y^2}{r^2}} \tag{4}$$

$$G_n(x,y) = \frac{j_v(x) + j_n(x)}{2} + \left(\frac{j_n(x) - j_v(x)}{2}\right) \cdot \sqrt{1 - \frac{y^2}{r^2}} \tag{5}$$

$$j_v(x) = \begin{cases} 0.412 \cdot \sqrt{1 - \frac{x^2}{0.461^2}}, & if\ 0 < x \le 0.406 \\ -1.66 \cdot x + 0.869, & if\ 0.406 < x \le 0.497 \\ -0.137 \cdot \sqrt{1 - \frac{(x - 0.667)^2}{0.186^2}} + 0.1, & if\ 0.497 < x \le 0.667 \end{cases} \tag{6}$$

$$j_n(x) = \begin{cases} 0.137 \cdot \sqrt{1 - \frac{x^2}{0.186^2}}, & if\ 0 < x \le 0.17 \\ -1.66 \cdot x + 0.338, & if\ 0.17 < x \le 0.261 \\ -0.412 \cdot \sqrt{1 - \frac{(x - 0.667)^2}{0.461^2}} + 0.1, & if\ 0.261 < x \le 0.667 \end{cases} \tag{7}$$

$$G_v(x,y) = \frac{j_v(x) + j_n(x)}{2} + \left(\frac{j_v(x) - j_n(x)}{2}\right) \cdot \sqrt{1 - \frac{y^2}{0.186^2}} \tag{8}$$

$$G_n(x,y) = \frac{j_v(x) + j_n(x)}{2} + \left(\frac{j_n(x) - j_v(x)}{2}\right) \cdot \sqrt{1 - \frac{y^2}{0.186^2}} \tag{9}$$

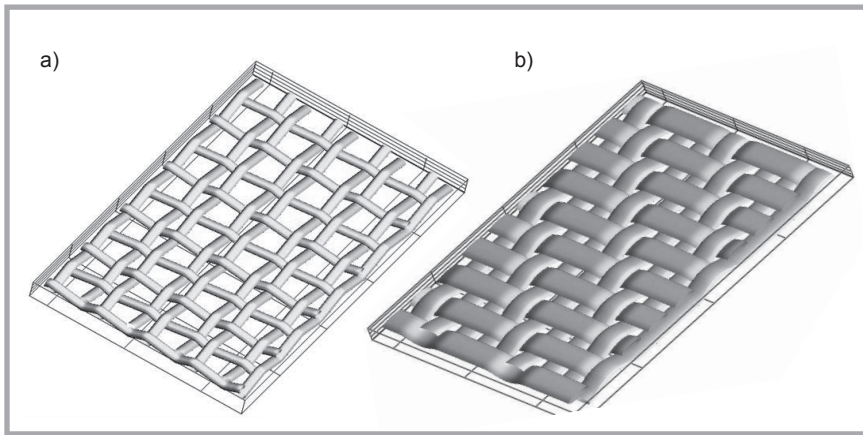***Equations (2), (3), (4), (5), (6), (7), (8)*** *and **(9)**.*

Figure 2. *Fabric 3D model: a) cotton fabric of plain structure with variable density, b) carbon fabric with twill structure).*

Simulation of the thread form with variable parameters

The effect of thread horizontal diameters increasing at the point of contact with the opposite thread system can be described by simulating the thread form in the fabric.

We introduce a parameter that determines the change in size of the thread cross-section in the horizontal direction r_n along the length of the simulation sample:

$$r_n = f_1(X_n) \qquad (10)$$

Where, $f_1(X_n)$ – functional dependence relating the coordinate along the length of the bending thread and the cross-sectional size of the thread in the horizontal direction.

It may happen that the cross-section's form changes along the length of the bending thread, i.e. at the contact point of the threads, it resembles the form of an ellipse more, and it tends to take a form close to a "stadium" or "oval" in the gap between threads. The equation of the reference section can be presented by an equation based on the ellipse equation and adding the degree LL to the argument:

$$y(x) = w\sqrt{1 - \frac{(x-v)^{2LL}}{r^2}} \qquad (11)$$

Where, w – vertical ellipse semiaxis; v – ellipse centre coordinate; r – horizontal ellipse semiaxis.
Where, the $LL = 1$ section has the form of an ellipse; when LL increases, the section form becomes more like a "stadium" and later like a rectangle (**Figure 3**).

The various cross-sectional forms of the thread can be obtained using **Equation (11)**. To do this, we set the argument LL_n as varying along the length of the simulated thread sample in the fabric:

$$LL_n = f_2(X_n) \qquad (12)$$

Where, $f_2(X_n)$ – functional dependence relating the coordinate along the length of the bending thread and the section form.

The smooth junctions from one form to another can be set with the help of the various algorithms.

To reflect the lateral bending of the thread in the fabric, we introduce the additional dependence $f_3(X_n)$.

The following variables must be specified for the function parametric definition that describes the three-dimensional model of yarn in a fabric: X_n, $Y_{n,m}$, $Z_{n,m}$. (n and m – the current number of the segment by which we divide the simulated yarn segment horizontally and vertically).

$$X_n = lo \cdot \frac{n}{N} \qquad (13)$$

Where, lo – simulated element length along the "X" axis; N – number of split intervals.

Then,

$$Y_{n,m} = f(X_n) - r_n + \frac{2mr}{N} \qquad (14)$$

Then, the general equation of the surface in a parametric form that describes the
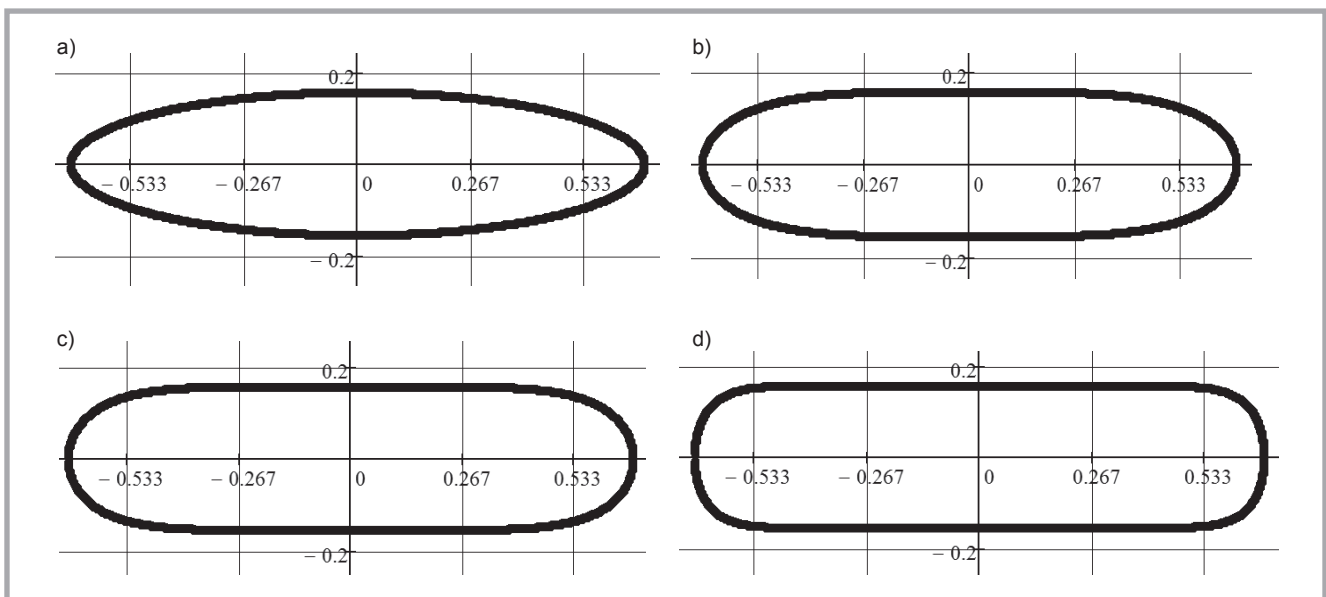

Figure 3. *Thread cross-section form: a) LL = 1, b) LL = 2, c) LL = 3, d) LL = 6.*
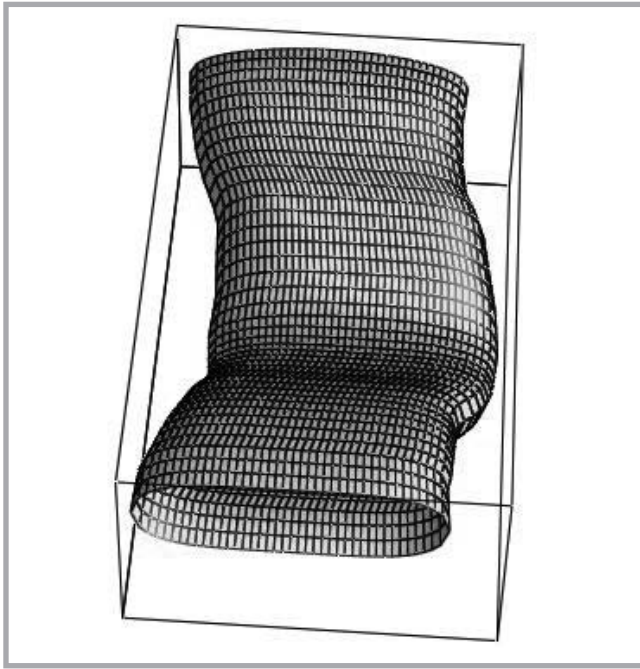
Figure 4. *Thread three-dimensional model for the fabric with the form change.*
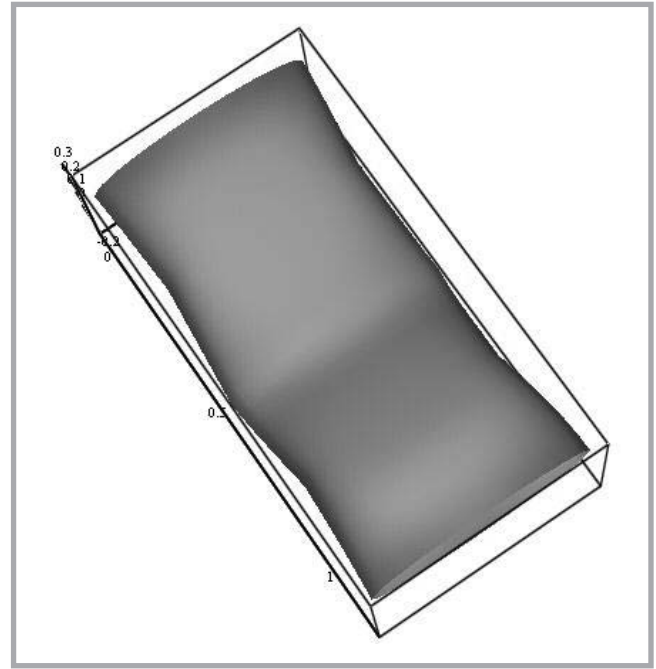

Figure 5. *Thread three-dimensional model for the fabric with the change in the size of the cross-section.*

lower part of the thread ($ZN_{n,m}$) in the fabric, taking into account lateral bending as well as changes in cross-sectional sizes and the form of the thread in the length, is given as ***Equation (15)***.

For the upper part ($ZV_{n,m}$) the equation is written as ***Equation (16)***.

In the case of cross-sectional forms other than the ellipse, ***Equations (1)***, ***(6)*** and ***(7)*** are different.

Equations (15) and ***(16)*** can be presented in an explicit form. For example, the general surface equations of the yarn lower part in an explicit form are given as ***Equation (17)***.

Assuming that the piecewise continuous function consists of two curves ($y_1(X)$, $y_2(X)$) and a straight line that connects them ($y = kX + b$), then the slope of the tangent *(k)* to the curves is determined as the derivative of function y_1 or y_2 at the tangency points, i.e.:

$$k = \dot{y}_1(x_1), \quad k = \dot{y}_2(x_2) \qquad (18)$$

Where, x_1 – x-coordinate of the first tangency point; x_2 – x-coordinate of the second tangency point.

To find parameters x_1, x_2, k & b that define the piecewise continuous function, there is a system of ***Equations (19)***:

$$\begin{cases} \dot{y}_1(x_1) = \dot{y}_2(x_2) \\ \dot{y}_1(x_1) \cdot x_1 + b = y_1(x_1) \\ \dot{y}_2(x_2) \cdot x_2 + b = y_2(x_2) \end{cases} \qquad (19)$$

The numerical solution of this system can be implemented by various software tools. After system solution (19), the pivot points and equation of the straight line are determined.

Based on ***Equations (13)-(16)***, taking into account the values contained therein, an algorithm for the construction of a fabric three-dimensional model was developed and implemented in a software form. The simulation results are presented in ***Figures 4*** and ***5***.

The thread three-dimensional model for the fabric taking into account lateral bending and changes in the form of the thread in the length is presented in ***Figure 4***.

The thread three-dimensional model for the fabric with the change in the size of the thread cross-section is presented in ***Figure 5***.

◼ Conclusions

The results obtained allow to state that the technique developed for fabric simulation creates models taking into account most of the characteristics of thread behaviour in fabric. The further development of this direction is the simulation of changes in the form and size of threads associated with their unevenness. In this case, the use of data on defect distribution in the yarn of different raw materials and varieties is possible.

Several types of equations are developed – in parametric and explicit form. Each form may have its own advantages in some cases. In our opinion, the piecewise

$$ZN_{n,m} = \frac{j_n(X_n) + j_v(X_n)}{2} + \frac{j_v(X_n) - j_n(X_n)}{2} \sqrt{1 - \left(\frac{Y_{n,m} - f_3(X_n)}{f_1(X_n)}\right)^{2 \cdot f_2(X_n)}} \qquad (15)$$

$$ZV_{n,m} = \frac{j_n(X_n) + j_v(X_n)}{2} + \frac{j_n(X_n) - j_v(X_n)}{2} \sqrt{1 - \left(\frac{Y_{n,m} - f_3(X_n)}{f_1(X_n)}\right)^{2 \cdot f_2(X_n)}} \qquad (16)$$

$$ZN(x, y) = \frac{j_n(x) + j_v(x)}{2} + \frac{j_v(x) - j_n(x)}{2} \sqrt{1 - \left(\frac{y - f(x)}{f_1(x)}\right)^{2 \cdot f_2(x)}} \qquad (17)$$

Equations (15), (16) and ***(17).***

function together with the equation in the explicit form allows to eliminate interpenetration between yarns in fabric as well as to simplify the model and conduct direct calculations of the coordinates of surface points.

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