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Probabilistic Model of Dynamic Forces in Thread in the Knitting Zone of Weft Knitting Machines, Allowing for the Heterogeneity of Visco-Elasticity Yarn Properties

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Abstract
This study presents an explanation of the stochastic character of dynamic thread loads in the knitting zone of weft-knitting machines based on a probabilistic model of the knitting process in which thread has been treated as a body of heterogeneous viscoelastic properties. Computer simulations were carried out according to the model presented, proving the influence of the randomly changing rheological parameters of thread on the force dispersion in thread in the knitting zone. Besides this it was established that the size of this dispersion also depends on the profile of the cam in the knitting zone.

Key words: *knitting zone, rheological parameters of thread, friction model, model of knitting process.*

Designations used:

F_0 - *preliminary thread tension before the knitting zone, in cN;*
F - *thread tension, in cN;*
F_A - *force in the take down zone, in cN;*
C, C_1 - *relative coefficients of tensile rigidity for the Zener model (Figure 1); the average values of these coefficients are accepted as input data in the calculation algorithm, in cN;*
$C_1, C_2, C_3, \ldots, C_n$ - *relative coefficients of the tensile rigidity C, in cN, of subsequent thread segments, for the branch of the Zener model representing elasticity after random modification,*
$C_{11}, C_{12}, C_{13}, \ldots, C_{1n}$ - *relative coefficients of the tensile rigidity C_1, in cN, of subsequent thread segments, for the branch of the Zener model representing visco-elasticity after random modification,*
V - *coefficient of variation of the relative coefficients of the tensile rigidity C and C_1 in the Zener model accepted for calculations b, in %;*
η - *relative coefficients of the dynamic viscosity, in cNs;*
ε - *relative elongation, in %;*
t - *time, in s;*
$v_\varepsilon = d\varepsilon/dt$ - *relative deformation speed of the thread,*
L_0 - *length of the thread link, in mm;*
z - *sinking depth of the needles, in mm;*
v_c - *linear speed of the cylinder, in m/s;*

Geometrical parameters of the knitting zone

d_h - *needle hook diameter, in mm;*
p - *sinker thickness, in mm;*
t_u - *needle pitch, in mm;*
γ - *angle of the knocking-off of needles, in °;*
β - *angle of clearing needles, in °;*
β_p - *angle of sinkers in the Relanit technique, in °;*
γ_p - *clearing angle of sinkers in the Relanit technique, in °;*
R_i - *quotient of the curve radius of cams in the knitting zone for the needles and needle pitch, in mm;*
R_p - *quotient of the curve radius of cams in the knitting zone for the sinkers and needle pitch, in mm;*
x_F - *length of the needle shank, in mm;*
x_{FP} - *length of the sinker shank, in mm;*
x_K - *co-ordinate of the clearing point for needles, in mm;*
x_{KP} - *co-ordinate of the clearing point for sinkers, in mm;*
α_p - *angle of thread feeding, in °;*
wt - *coefficient of pitch take-up.*

Parameters of thread for the knitting model

μ - *conventional friction coefficient of thread against forming elements for the initial position of needles and sinkers,*
η_i, η_p - *relative dynamic viscosity of thread pulled through the hooks of needles η_i and sinker edges η_p in the Zener model,*
a_i, a_p, n_i, n_p. - *coefficients in the generalised principle of the friction of thread pulled through the hooks of needles (a_i, n_i) and sinkers (a_p, n_p),*
d - *yarn diameter, in mm.*

Introduction

During the knitting process, tension zones can be observed, which are situated between the guides and on their surface in the yarn feeding zone, as well as in the knitting zone between the loop forming elements. One of the basic parameters characterising the knitting process is the force in the thread, which determines the efficiency of the knitting machine and the quality of the fabric produced.

The changing thread tension in the knitting process results from the production technology and factors connected with the heterogeneous mechanical properties of the yarn. In the modelling of the forces in the thread that is presented by different authors [1 - 10], the aspect of force variability, connected with the heterogeneous mechanical properties of yarn, is usually neglected. The values of forces in the thread determined according to these models do not fully reflect the real phenomena taking place while the thread is moved through the frictional barriers or what is happening in the knitting zone. The modelling results do not determine the dispersion of force values, which accompanies the real processes of turning yarn into a knitting fabric.

The authors in their earlier works [11 - 13] worked out a model comprising the influence of random changes in thread properties over short sections on forces generating in the thread as it is moved through the drawing zone and on the characteristics of these forces. They also worked out a probabilistic model of the process of moving the thread through frictional barriers [12, 14]. In the first models only

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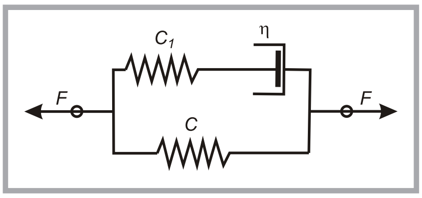


***Figure 1.** Three-element Zener model, C, C_1 – relative coefficients of tensile rigidity,* η *- viscosity.*

the elastic properties of the thread were taken into account [11], but the models constructed later allowed for viscoelastic properties [12 - 14]. After experimental verification of the models described, allowing for the heterogenous character of the mechanical properties of the thread, the mathematical model of the knitting process was modified [5 - 8] based on introducing probabilistic models, allowing for randomly changing values of the rheological parameters of the thread.

Physical basis for considerations concerning thread properties

The research on the dynamic properties of thread and fibers shows that yarn should be considered as a body of viscoelastic properties. The thread's behaviour during dynamic stretching, force relaxation and creep can be presented by means of a Zener three -element rheological model (***Figure 1***) [16].

The Zener model consists of two parallel branches. The first one represents elastic properties causing deformation directly proportional to the force. These properties are characterised by the coefficient of relative tensile rigidity *C* in cN. The second branch represents visco-elastic properties characterising the coefficient of tensile rigidity C_1 in cN and viscosity coefficient C_1 in cN.

The dependence between the deformation ε, tension force *F*, time *t* of the force action and rheological parameters of the Zener model *C*, C_1 & η is described by the differential equation [1, 6, 8]:

$$F + \frac{\eta}{C_1} \cdot \frac{\mathrm{d}F}{\mathrm{d}t} = C \cdot \varepsilon + \left(C + C_1\right) \cdot \frac{\eta}{C_1} \cdot \frac{\mathrm{d}\varepsilon}{\mathrm{d}t} \quad (1)$$

In cases where the deformation speed dε/d*t* = *const* = v_ε and relative deformation

$$\varepsilon = \mathrm{d}\varepsilon/\mathrm{d}t \cdot t = v_\varepsilon \cdot t \quad (2)$$

(tension change at a constant deformation speed) the solution of equation (1) is the dependence:

$$F = F_0 \cdot e^{\frac{-t \cdot C_1}{\eta}} + C \cdot \varepsilon + \eta \cdot v_\varepsilon \cdot \left(1 - e^{\frac{-t \cdot C_1}{\eta}}\right) \quad (3)$$

Dependence (3) describes changes in thread tension when the thread is stretched at a constant speed of growing relative deformation. In the calculations made so far, for instance in work [8], the values of coefficients *C*, C_1 and η, which were material constants, remained constant along the whole thread transported through the drawing zone. Thus, the tension values received from the formulas presented above were expected, were average values and gave no information as to the tension variability observed during the experiments.

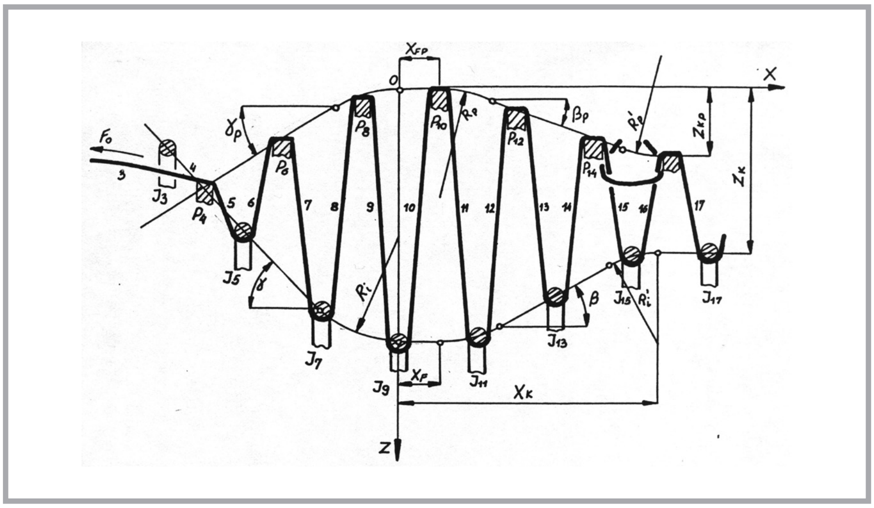


***Figure 2.** General geometrical model of the knitting zone on weft-knitting machines.*

Model of the knitting process

Preliminary consideration of a new model of the knitting process were published in [6, 13]. The main assumptions of the knitting process established in works [1, 8] were:

- thread is a material of viscoelastic properties, in which the relation between the relative elongation ε, tension force *F* and time *t* of the force action is described by the Zener three-element rheological model,
- the relation between the force before and after the frictional barrier is described by dependence (3), and the general friction law $T = a N^n$.

As for the geometry of the system, it was assumed that:

- thread sections between the loop forming elements form straight segments,
- the axis of the thread at the contact point is parallel to the curvature of the friction barriers,
- the cams guide the needles in the knitting zone.

Geometrical parameters of the knitting zone, taken into account in the model presented in works [1, 8], are shown in ***Figure 2***. Moreover, this model also allows for the parameters of the thread and of the knitting process.

If angles γ_p and β_p equal zero, then we have a classic knitting zone. In a classic knitting zone, the knocking-off sinkers do not move vertically in a reciprocating motion but remain all the time at the same height. These are stitch cams of point and linear sinking depth.

The following thread parameters were taken into account in the knitting zone model [1, 8]:_μ, η_i, η_p, C, C_1, a_i, a_p, n_i, n_p, d.

The listen below technological parameters of the knitting process were alo taken into account in the model: F_o, F_A, z, v_c.

The model allows for all the most important parameters of the knitting process. One calculation loop of the programme refers to calculations made after shifting the cylinder of the warp-knitting machine by Δx. The main axis of calculating the temporary course of forces in the thread and the length of the taken-up thread takes into consideration the equilibrium conditions of dynamic forces in the thread on individual frictional barriers after the cylinder has shifted by Δx.

Sample results of the knitting process simulation received from this algorithm arepresented in ***Figure 3***.

Assumptions and theoretical basis for considering the knitting process, taking into account the heterogeneous mechanical properties of the thread used

The model presented in ***Figure 4*** [6] does not allow for the random heterogeneity of rheological properties observed along the thread. The calculation results do not make it possible to assess the variability of thread tensions in the knitting zone.

The authors of the work modified the model of the knitting process from [1, 8], taking into account the random heterogeneity of thread properties. They applied the probabilistic model of drawing the thread through frictional barriers, worked out in [12, 14, 15], to the old model. In the numerical stimulations carried out, coefficients C and C_1 underwent random modifications.

For that purpose the following assumptions were made:

- all the assumptions of the knitting process model formulated in [6, 13] are valid, and:
  - the thread consists of short segments (links). The properties of each of them can be described by the Zener model with the use of different coefficients: C in cN - C_1, C_2, C_3,... C_n, C_{1n} in cN - C_{11}, C_{12}, C_{13}... C_{1n}, and the viscosity η,
  - while forming one loop, the rheological properties remain the same, which means that one loop is formed of one elementary link of the thread (***Figure 4.b***),
  - each subsequent repetition of the calculation cycle of forming a single loop is based on different, randomly modified values of coefficients C and C_1, in which the mechanical properties are determined using the Zener model,
  - values of coefficients C and C_1 for subsequent loops change randomly and have a normal distribution,
  - before the calculations, one has to determine:
- average values of C in cN and C_1 in cN,

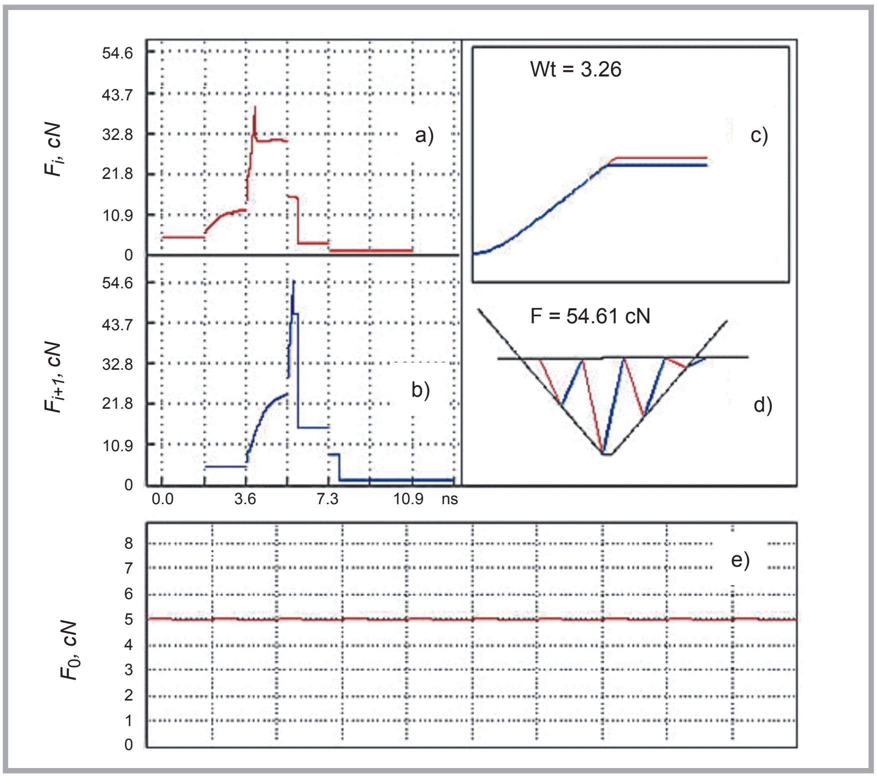


Figure 3. Results of the digital simulation of the knitting process in the form of a temporal course of dynamic forces in the thread and escalation curves of the coefficient of take-up for the pitch of individual segments of the loop formed: $\beta = 50°$, $x_F = 0.4$ mm, and $z = 4.3$ mm; a) value of the force on the right side of the sinker, b) value of the force on the left side of the sinker, c) growing coefficient of take-up for the pitch, d) diagram of the knitting zone, e) values of forces in the thread in the feeding zone.

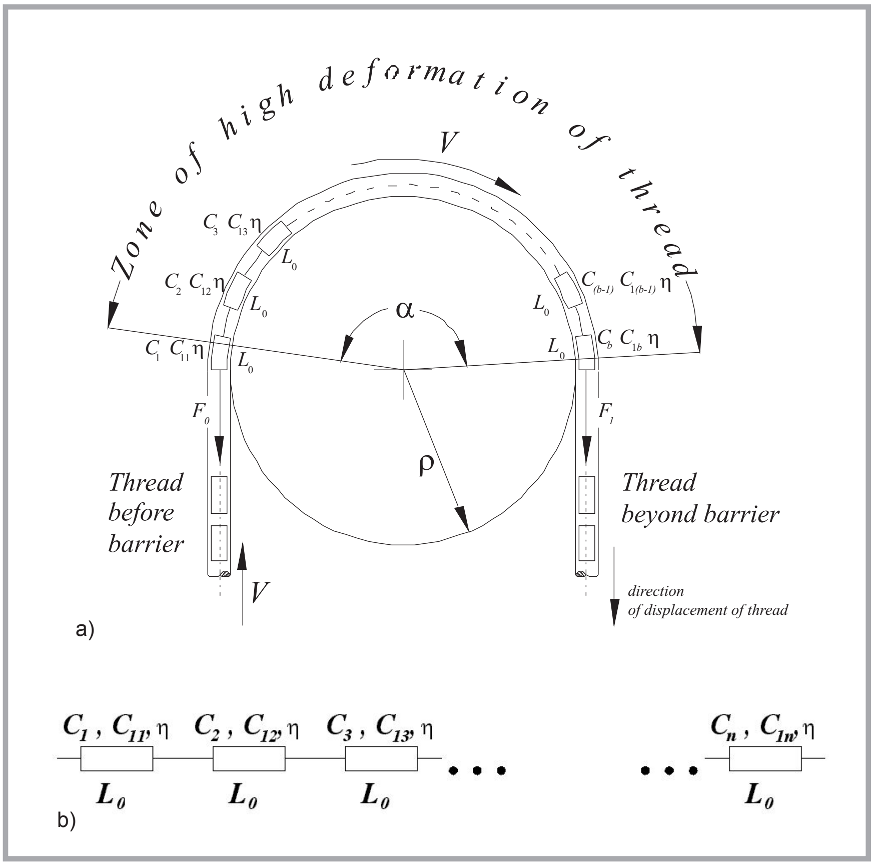


Figure 4. Assumed rheological model of thread with heterogeneous properties; a) thread model, b) a model of thread on a frictional barrier.

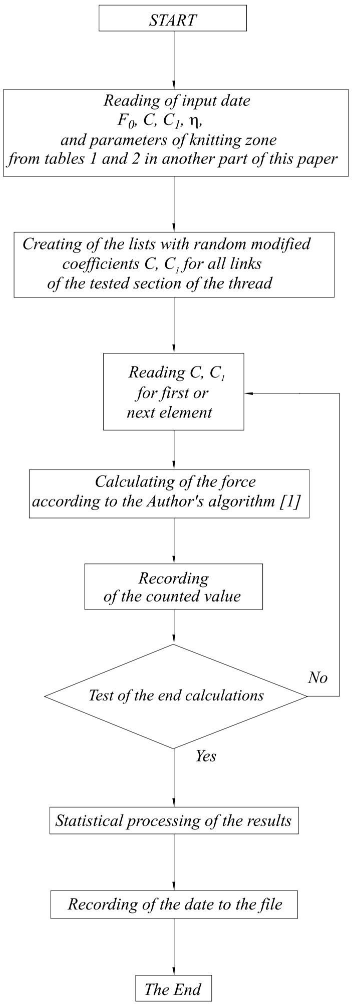
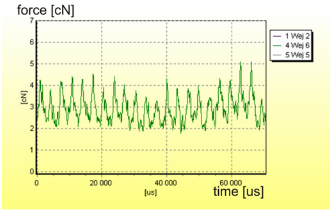
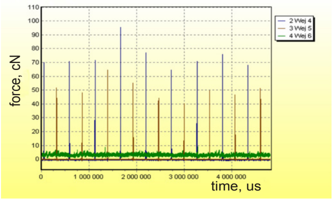


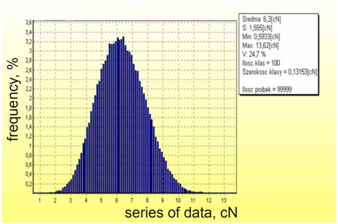
__Figure 5.__ Modified algorithm for calculating forces in thread in the knitting zone accepted for our considerations.



Measurement results of preliminary tension F_0



Measurement results of thread tension in the knitting zone: F_i, F_{i+1}



Histogram and statistical parameters of the measured thread tensions.

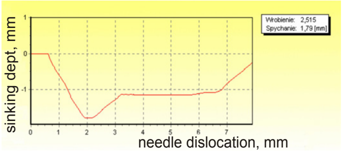


Diagram of the measurements of sinking depth and take-up.

__Figure 6.__ General view of a computer measuring weft-knitting machine with some measurment results, which have been displayed in order to show how they are worked out and presented by the data acquisition system of the weft-knitting machine.

- the coefficient of variation for *C* and C_1,
- the number *k* of thread links (knitted loops) for which calculations will be made,
- geometrical parameters of the knitting zone,
- technological parameters of the process.

Random modification of the average values of C and C_1 is based on their computer processing, which allows for the average value and coefficient of variation expected. This processing is independent and different for each of the coefficients. Common properties are the coefficient of variation and normal distribution. The result of this operation are lists of values of coefficients *C* and C_1 which are successively taken for calculations, according to the algorithm.

Calculation algorithm

The results received by the authors from the algorithm presented in ***Figure 5*** prove that the model of the knitting process [1, 8] supplemented with a discrete probabilistic model of drawing the thread through frictional barriers [12 - 15] makes it possible to generate a stochastic character of the maximum forces in the thread in the knitting zone.

Practical verification of the calculation results was carried out with the use of a computer measuring weft-knitting machine designed and constructed within research projects [17 - 19]. Experimental verification was limited to the case when the knitting process was carried out with the use of a stitch cam of linear sinking depth, which excludes the phenomenon of re-drawing the thread within the knitting zone.

Experimental verification of the model using a special measuring weft-knitting machine

Description of the machine its measuring and interpretation possibilities methods. A computer measuring weft-knitting machine was used for experimental verification of the model established. This weft-knitting machine makes it possible to measure forces in the knitting process using equal voltage or an equal segment way of yarn feeding, different lengths of the thread section in the feeding zone, recti- and curvilinear cam profiles and different values of the pitch run-in ratio.

The diameter of the weft-knitting machine is φ =4”E and the needle pitch 14E. The machine is equipped with a set of necessary converters and measuring systems [17].

Data (except for the mass of yarn dust) are acquired on-line by means of a computer system of data acquisition and analysis. Apart from data acquisition, the software makes it possible to mathematically process the data (statistical and harmonic analysis) and visualise both the values measured and the calculations results. Data analysis refers to both momentary and top values. The weft-knitting machine discussed and some measurement results are presented in ***Figure 6***.

The free vibration frequency of the measuring transducers equals 15.9 kHz, which makes it possible to measure forces in the thread in the knitting zone without amplitude distortions. The system used makes it possible to measure dynamic forces in the thread in the knitting zone during normal operation of the weft-knitting machine with holding down-knocking over sinkers.

Experimental verification of the model established

Computer simulation of dynamic forces in the thread in the knitting zone was carried out for the input data presented in ***Table 1*** and ***2***.

All simulations were conducted for:

- A cam of linear sinking depth x_F = 7.2 mm (***Figure 7,*** see page 66), for which no thread re-drawing can be observed (denotation 50°/7.2/50°),
- A cam of point sinking depth characterised by a knocking-off angle of 50°, and a clearing angle of the needles after the loop is formed of 30° (denotation 50°/30°),
- A cam of the point sinking depth characterised by a knocking-off angle of 50° and clearing angle of the needles after the loop is formed of 50° (denotation 50°/50°),

Table 1. *Input data for the computer simulation. Geometrical parameters of the knitting zone.*

Geometrical parameters of the knitting zone									Parameters of the knitting process	
d_h	p	t_u	x_F	x_{FP}	γ	$γ_p$	R_i	R_p	F_a	v_c
mm					°		-		cN	m/s
0.5	0.35	1.81	7.2	0	50	50	0.25	0.25	3.0	0.7

Table 2. *Input data for the computer simulation. Thread parameters.*

d	m	$η_i$	$η_p$	C	C_1	a_i	a_p	n_i	n_p
mm	-	cN s		cN		-		-	
0.1	0.2	3	3	4200	3800	0.426	0.76	0.86	0.86

The calculation results are subsequent values of the maximum thread tension in the knitting zone. A sample diagram of maximum thread tensions in the knitting zone received from measurements made on the weft-knitting machine is presented in ***Figure 8.a*** (see page 66). The results generated numerically by means of the algorithm discussed above are presented in ***Figure 8.b*** (see page 66).

As shown in the histograms in ***Figure 9.b*** (see page 66), the maximum forces in the knitting zone received by means of simulation according to the probabilistic model of the knitting process established reflect the results of experimental research ***Figure 9.a*** (see page 66).

A comparison of the values of average tensions received as a function of the sinking depth for different coefficients of variation of the relative tensile rigidity C and C_1 is presented in ***Figure 10*** (see page 66). The red curve illustrates experimental data received for a cam of linear sinking depth, whereas other curves stand for theoretical data received from the calculation model presented.

Theoretical data was received as a result of numerical calculations carried out for different coefficients of variation equal to - 10, 15, 20, 25, 30, 35, 40, 45, 50 & 60% for the relative tensile rigidity of the branch of Zener model C and C_1. Three types of curves can be distinguished on the diagram in ***Figure 11*** (see page 66), each of which refers to a stitch cam of a different profile. The results of computer simulation prove that the influence of the value of the coefficient of variation for the tensile rigidity C and C_1 on the average values of the maximum thread tension in the knitting zone is practically negligible. According to expectations confirmed by previous research [6], the lowest values of forces in the thread were received for the 50/50 cam, in which the phenomenon of thread re-drawing is the most intensive. However, for this cam the coefficient of variation of forces in the thread is the smallest (***Figure 11,*** see page 66). When the knocking-off angle γ equals the clearing angle β, the length of the thread released from the clearing needle equals the length of thread on the knocked-off needle required. Thus, in the case of thread re-drawing, the demand for thread by the knocked-off needle is compensated

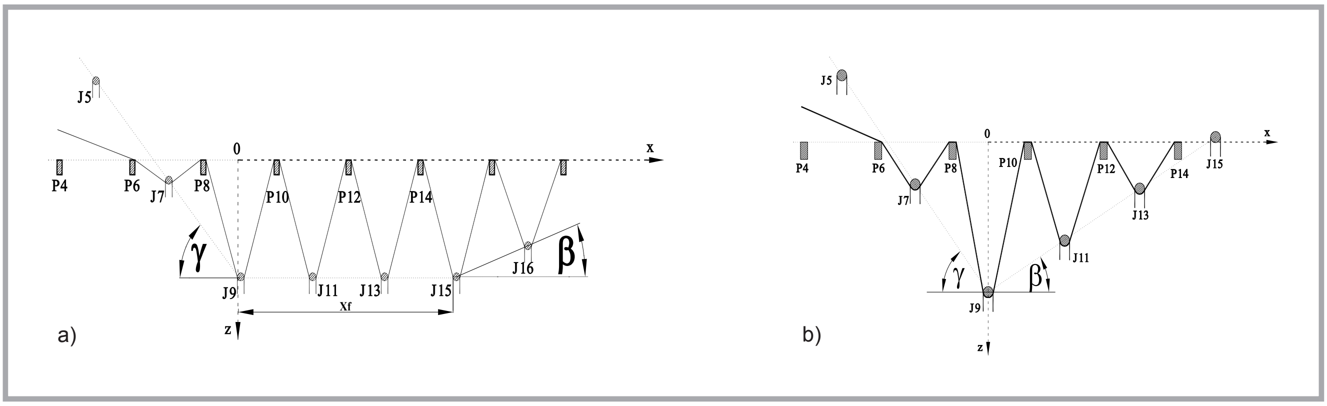


Figure 7. *Knitting zone diagram: a – for a cam of linear sinking depth x_F = 7.2 mm, knocking-off angle g, and clearing angle b; b – for a cam of point sinking depth and knocking-off angle g equal to clearing angle β.*

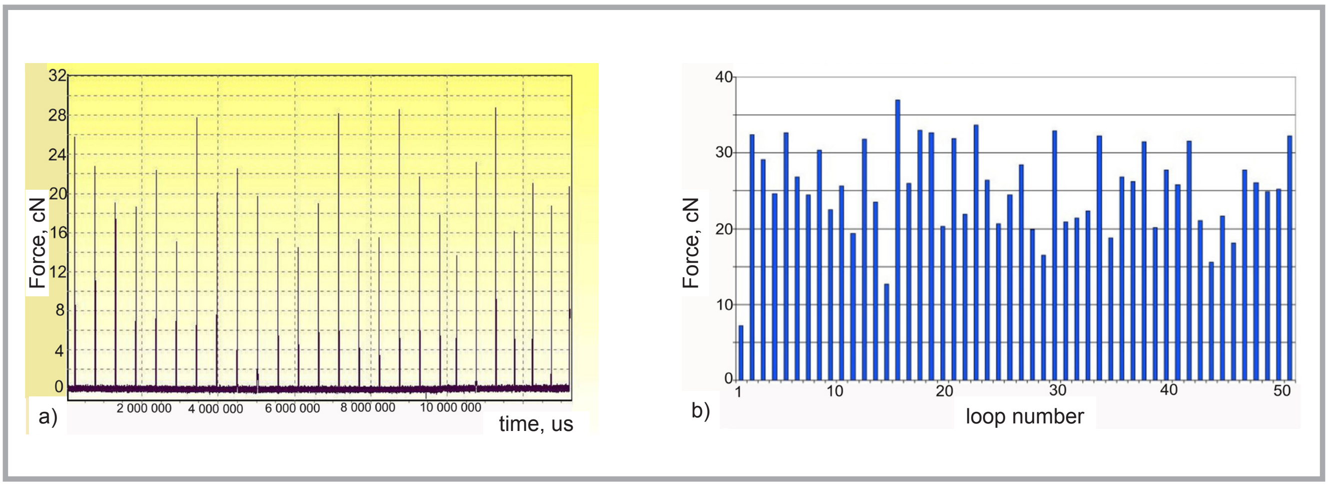


***Figure 8.** Maximum values of forces in the thread in the knitting zone: a – sample maximum values of forces in the knitting zone received during the experiment for a sinking depth of z = 1.5 mm, b- sample analytical maximum values of forces in the knitting zone for a sinking depth of z = 1.5 mm and coefficient of variation for C and* C_1 *V = 25%.*

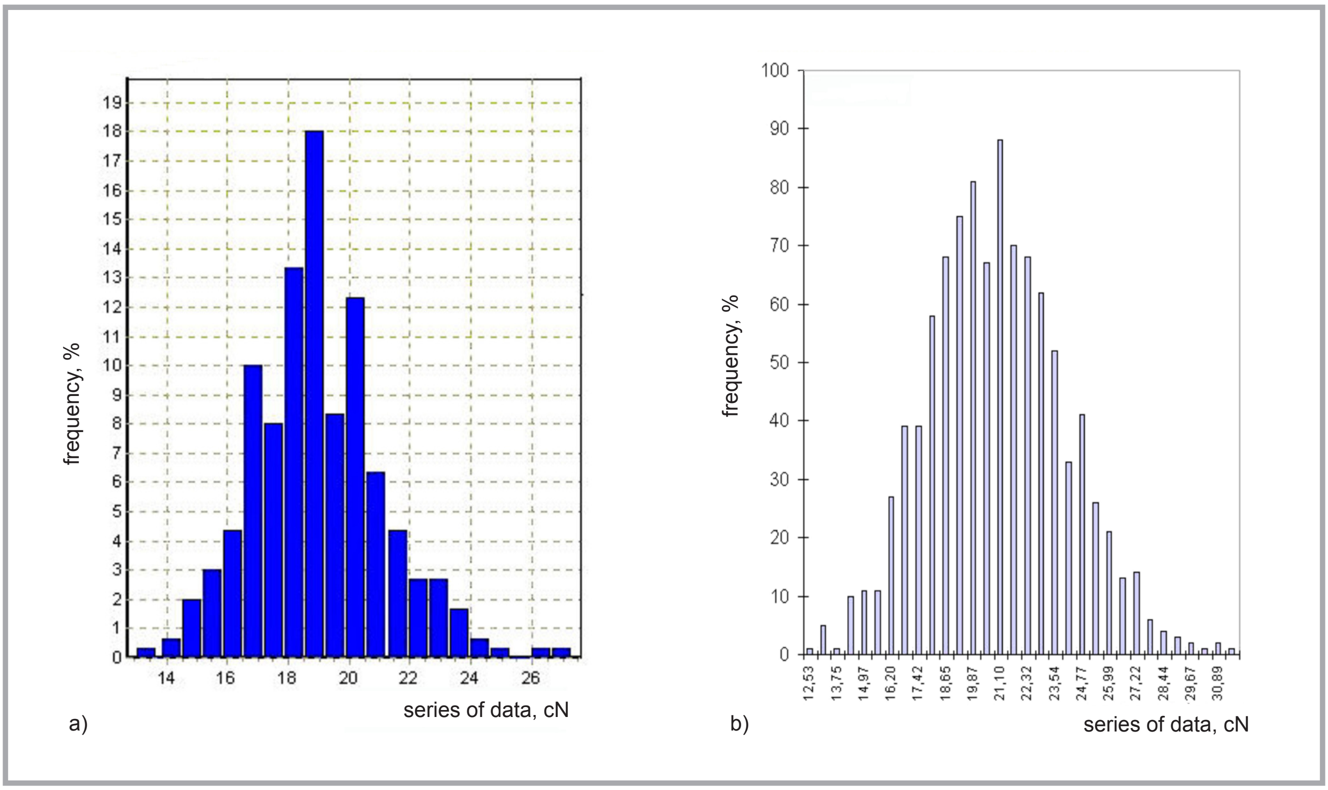


***Figure 9.** Histograms for tensions presented in Figsures 8.a and 8.b.*

by re-drawing, and the maximum force values in the knitting zone depend on the re-drawing forces. Low values of the coefficient of variation can be ascribed to the high repeatability and simultaneity of the thread re-drawing towards thread sections hanging on the knocked-off needle in the knitting zone

In case of a 50/30 cam, the time repeatability of the phenomenon of re-drawing the thread towards the thread sections hanging on the knocked-off needle, to a large extent, depends on the rheological parameters of the thread. Random values of the rheological parameters of the thread determine larger changes in the forces in the thread in the knitting zone.

In the case of a cam of linear sinking depth, the values of the coefficient of variation calculated for experimental average values of the maximum forces in the thread in the knitting zone point to their slight increase within the range of 13 - 16%, accompanied by an increased sinking depth and maximum forces in the thread in the knitting zone (***Figure 11***). The largest increase in the value of the coefficient of variation of forces in the thread in the knitting zone can be observed for the cam of point sinking depth 50/30.

Experimental verification carried out for a cam of linear sinking depth proved that the values of the coefficient of variation of the maximum forces in the thread in the knitting zone which are calculated and those from the measurements (red curve) are contained in the area determined by the curves from model calculations. It should be noted that the model curves shown in ***Figure 11*** were received for a determined value of the coefficient of variation for C and C_1 equal to 30 – 40%.

Conclusions

1. The model of the knitting process, supplemented with a discrete probabilistic model of the process of drawing thread through frictional barriers, makes it possible to model the stochastic character of the maximum forces in the thread in the knitting zone and explains one of the reasons for the changeability of forces in thread during the knitting process. The experiments confirm the results received during computer simulation.

2. The results of numerical simulation proved that an increase in the values of the coefficients of variation for rheological parameters like the relative tensile rigidity C and C_1, according to the three-element standard Zener model, only slightly affects the average values of forces in thread in the knitting zone.

3. An increase in the knocking-off depth of needles in the knitting zone is accompanied by an increase in the value of the coefficients of variation of maximum forces in the thread in the knitting zone, the intensity of which depends on the profile of the cam in the knitting zone.

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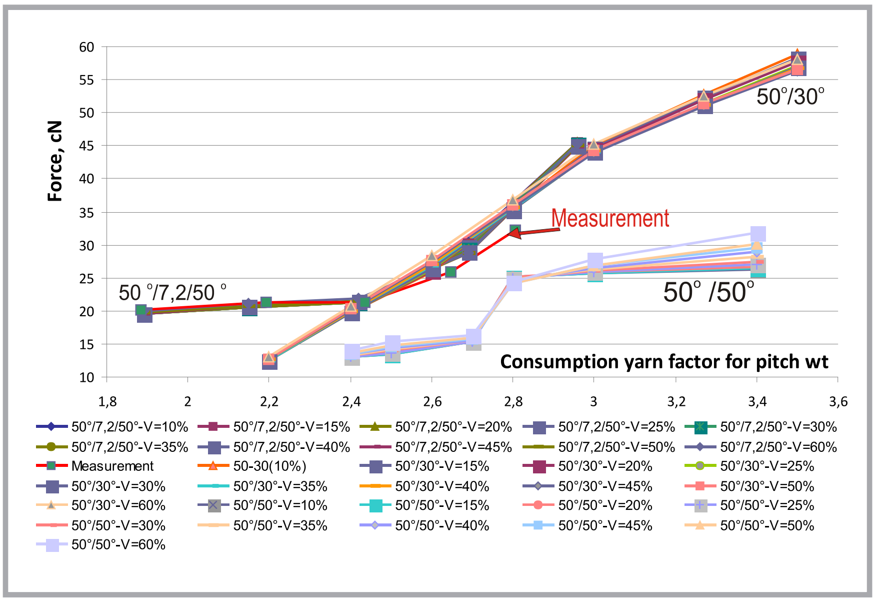

Figure 10. *Comparison of medium values of maximum thread tensions in the knitting zone received experimentally and as a result of computer simulation*

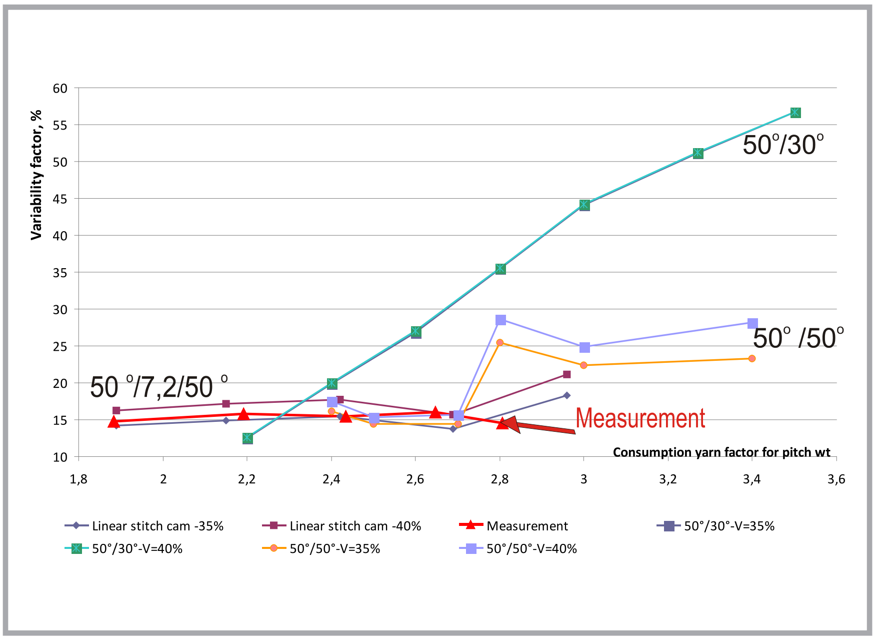

Figure 11. *Comparison of the values of coefficients of variation for maximum forces in the thread in the knitting zone received from measurements and simulations.*

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